MCMC algorithm for investigating variation in traffic flow

Toshiyuki Yamamoto
Nagoya University

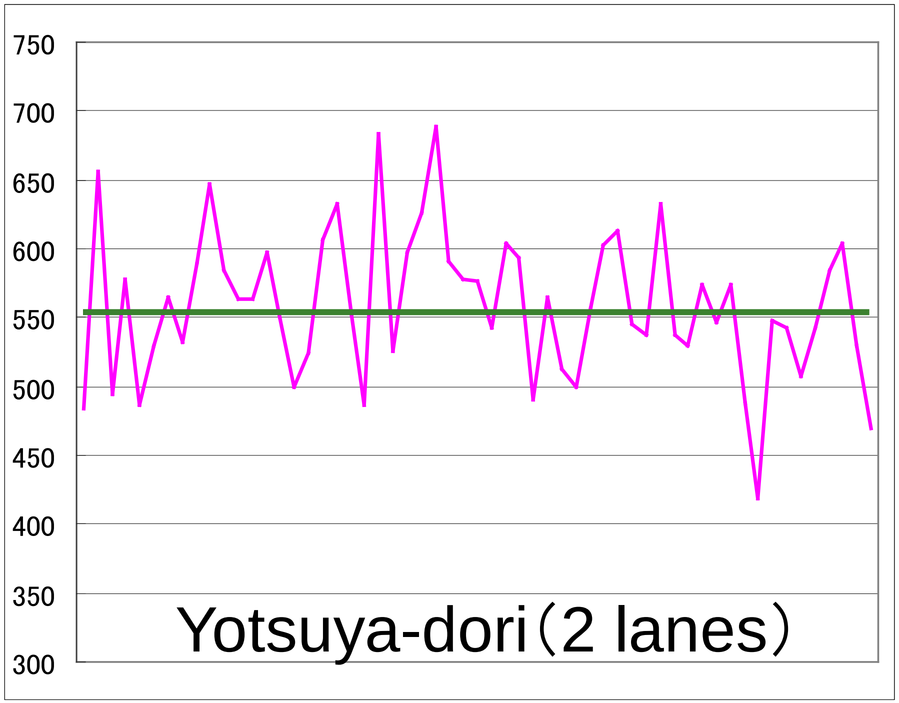
Introduction

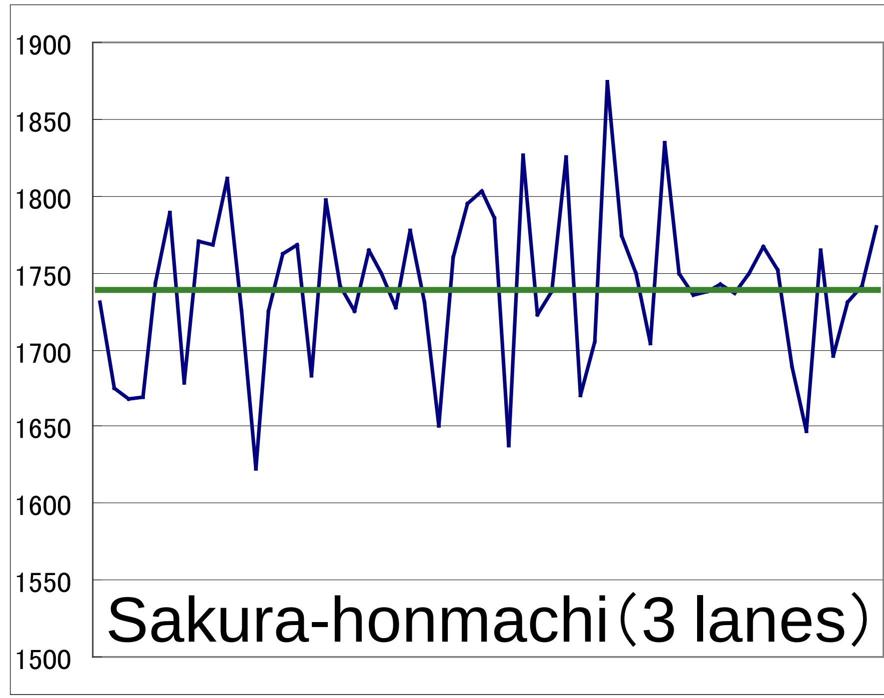
- Needs for complex traffic control and emerging driver information systems have led to increasing interests in:
  - Stochastic elements to account for errors in drivers' perceptions, and
  - Day-to-day variation in behaviour.

- Stochastic user equilibrium, despite the name, forms a fixed flow pattern, thus unable to represent variations in traffic flow.

Examples in reality

- Traffic counter data at inner city links
  - 2002/January~March, weekdays, 8:00~9:00





Average: 554.9, s.d.: 63.3

Average: 1740.7, s.d.: 50.5

Stochastic user equilibrium

- A traveller selects the route which he/she perceives to have minimum cost, including errors.
- The traveller chooses the route stochastically.
- Traffic flow results from the choices of the travellers, ~~so the flow should be stochastic.~~

  By The Weak Law of Large Numbers,

  the flow gets closer to a fixed pattern.

Conditional SUE by Hazelton (1996)

- A traveller selects the route which he/she perceives to have minimum cost, including errors.
- The traveller chooses the route stochastically.
- Traffic flow results from the choices of the travellers, so the flow is stochastic.

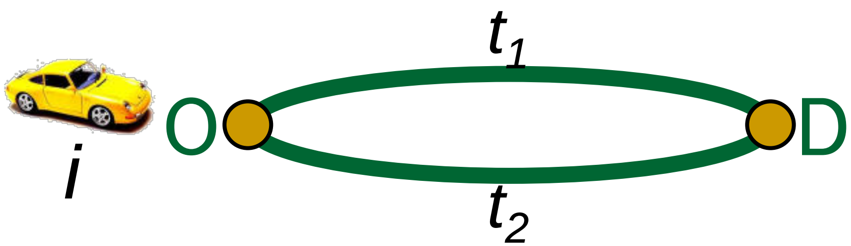
Markov chain Monte Carlo (MCMC) algorithm is used to solve the stochastic flow.

Objective of this study

By using Hazelton's Conditional SUE, variation of traffic flow is investigated in some cases.

- Variation of traffic flow:
  - Link flow
  - Link travel time
  - Link speed
- Cases:
  - Different demand, capacity, and scale parameter
  - 1 OD 3 routes with overlapping
  - Heterogeneous value of time

Traveller's stochastic choice



- Traveller's stochastic choice depends on route travel time: $Pr(R_i = r_1 | t_1, t_2)$
- Route travel time is a function of other travellers' choice: $t_k = F(r_1, r_2, \ldots, r_{i-1}, r_{i+1}, \ldots, r_N)$
- Thus, traveller's choice depends on other travellers' choice:

$$Pr(R_i = r_1 | R_1, R_2, \ldots, R_{i-1}, R_{i+1}, \ldots, R_N)$$

Stochastic traffic flow

- Distribution of stochastic flow is a function of travellers' choice: $Pr(R_1, R_2, \ldots, R_N)$.
- We only know $Pr(R_i=r_1 | R_1, R_2, \ldots, R_{i-1}, R_{i+1}, \ldots, R_N)$.

How to find $Pr(R_1, R_2, \ldots, R_N)$ from $Pr(R_i=r_1 | R_1, R_2, \ldots, R_{i-1}, R_{i+1}, \ldots, R_N)$?

MCMC algorithm samples state according to joint distribution using conditional distribution.

MCMC algorithm

(1) For the initial state, assign arbitrary ($R_1(0), \dots, R_N(0)$).

(2) Re-assign $R_1(j+1)$ probabilistically according to

$$Pr(R_1(j+1) | R_2(j), R_3(j), \dots, R_N(j))$$

$$R_2(j+1) \sim Pr(R_2(j+1) | R_1(j+1), R_3(j), \dots, R_N(j))$$

……

$$R_N(j+1) \sim Pr(R_N(j+1) | R_1(j+1), R_2(j+1), \dots, R_{N-1}(j+1))$$

(3) It is proved that the iterations of (2) reach to the probabilistic equilibrium state according to the joint distribution.

Functions used in this study

- Modified BPR function is used for link cost.

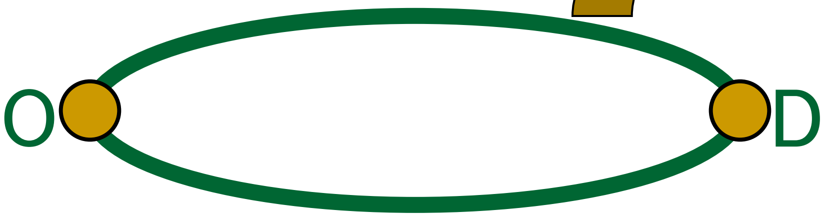
$$t(x) = l\left\{1 + 2.62\left(\frac{x}{C}\right)^5\right\}$$

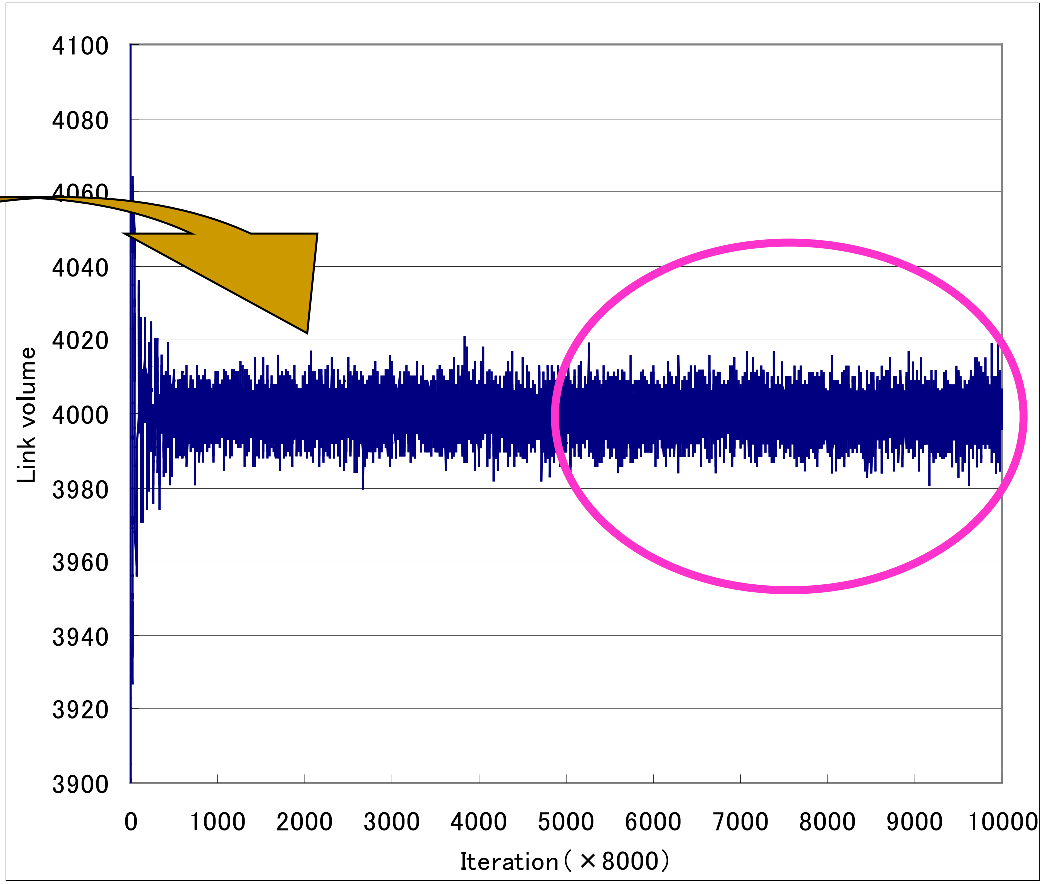
- Multinomial logit function is for route choice.

$$\Pr(R_i = r_k \mid R_{-i}) = \frac{\exp\{-\theta\, t(r_k \mid R_{-i})\}}{\sum_j \exp\{-\theta\, t(r_j \mid R_{-i})\}}$$

Example of simulation

- demand = 8000,
  θ = 0.5, l = 5km,
  C = 4000





Link volume

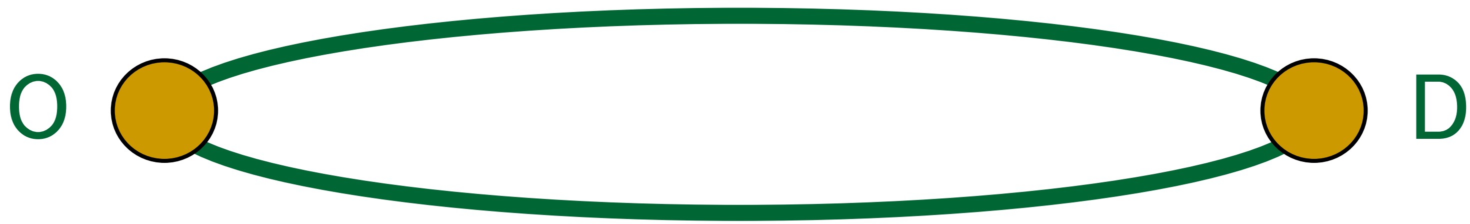
Average	Variance
4000	58.8

Iterations until convergence are discarded.

Interdependency of travellers' choice

Simple 1 OD 2 links

- $\theta = 0.5$, l = 5km, demand = 8000, C = 4000

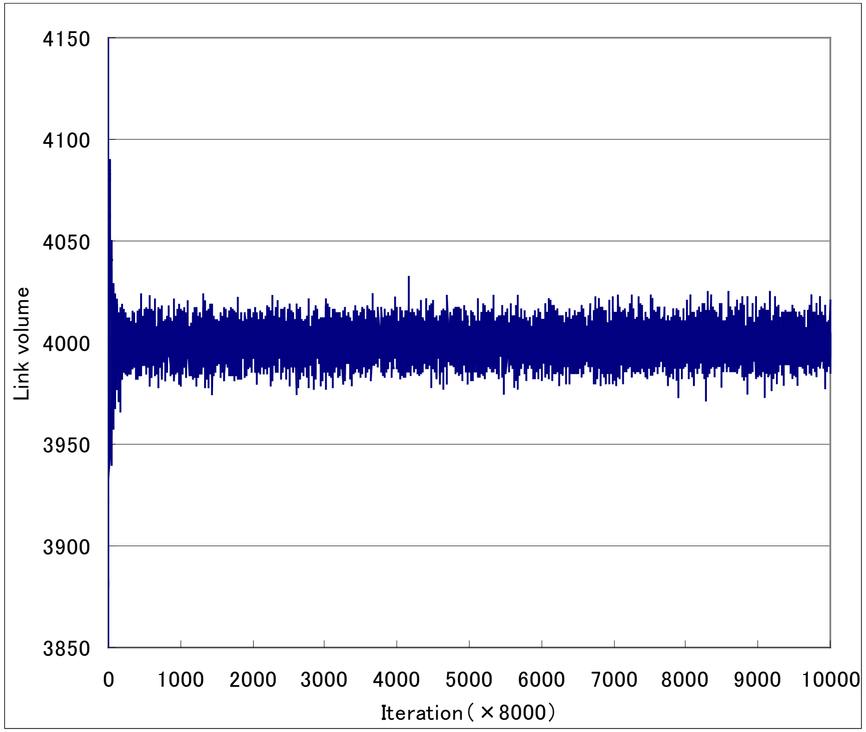


Comparison between

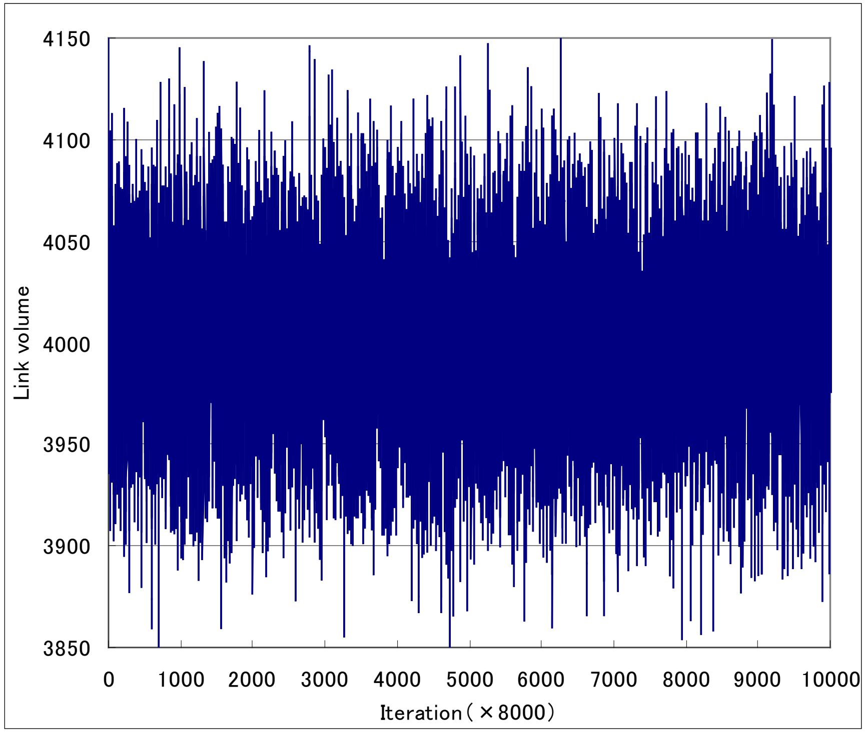
- $Pr(R_i{=}r_1 | R_1, R_2, \ldots, R_{i-1}, R_{i+1}, \ldots, R_N)$, and
- Independent choice: $Pr(R_i{=}r_1 | E(t_1), E(t_2))$

Interdependency of travellers' choice

Inter-dependent choice

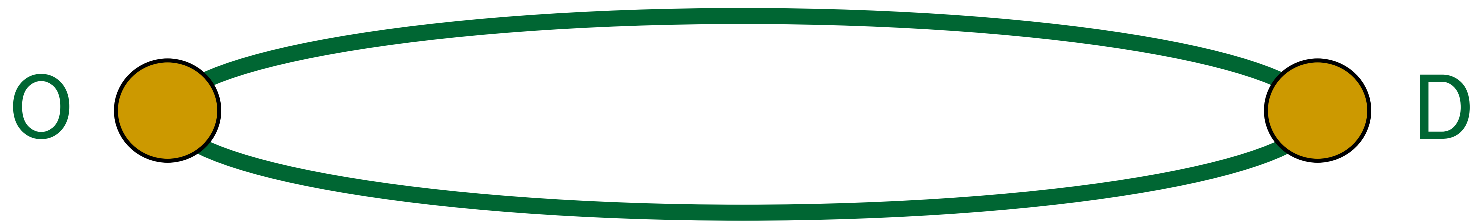


Independent choice



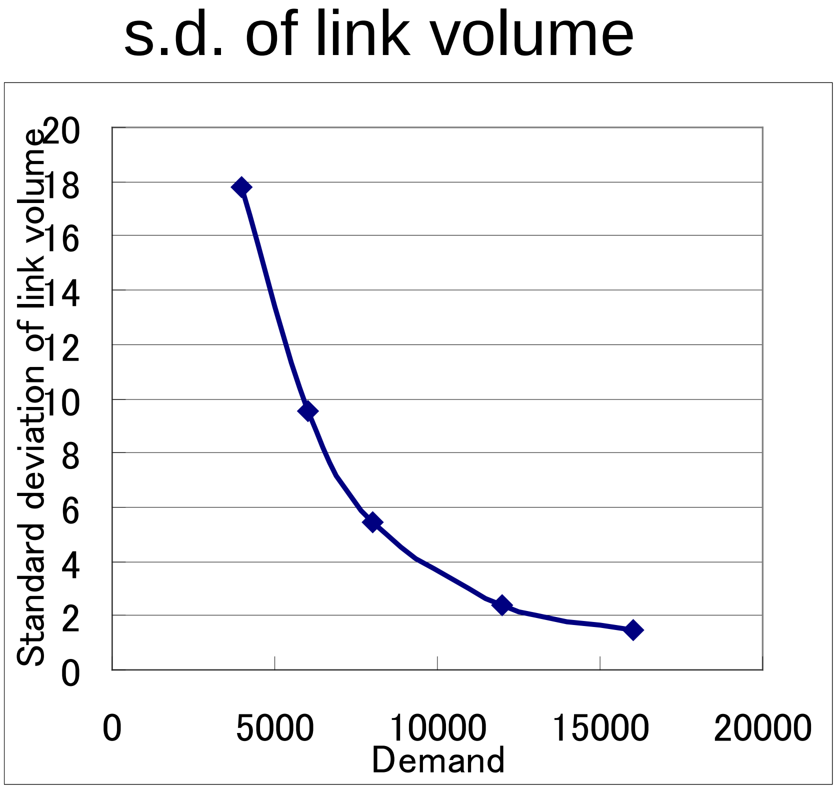
- Over-estimate of variation if independence is assumed.

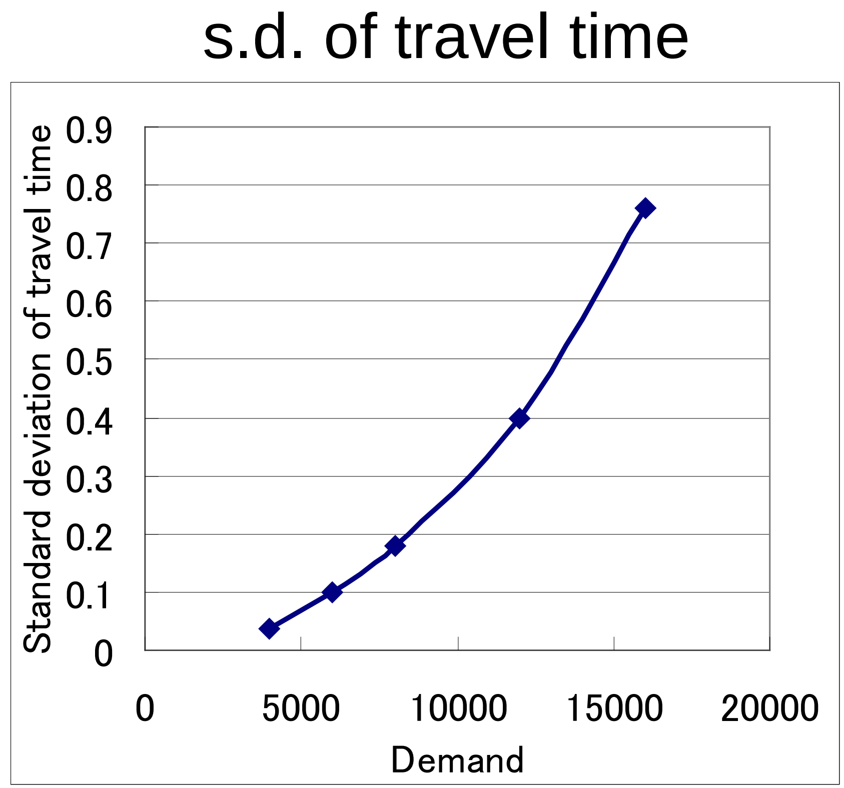
Case 1: Effects of demand, capacity, and scale parameter



	Lower bound		Base case		Upper bound
Demand	4000	～	8000	～	16000
C	2000	～	4000	～	8000
l (km)			5		
θ (1/min)	0.1	～	0.5	～	1

Effect of demand

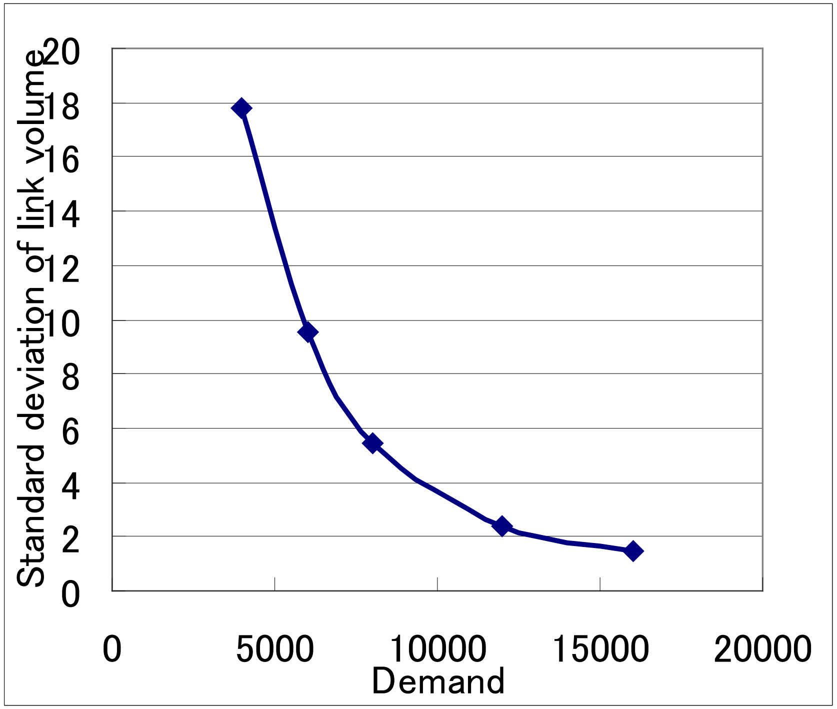




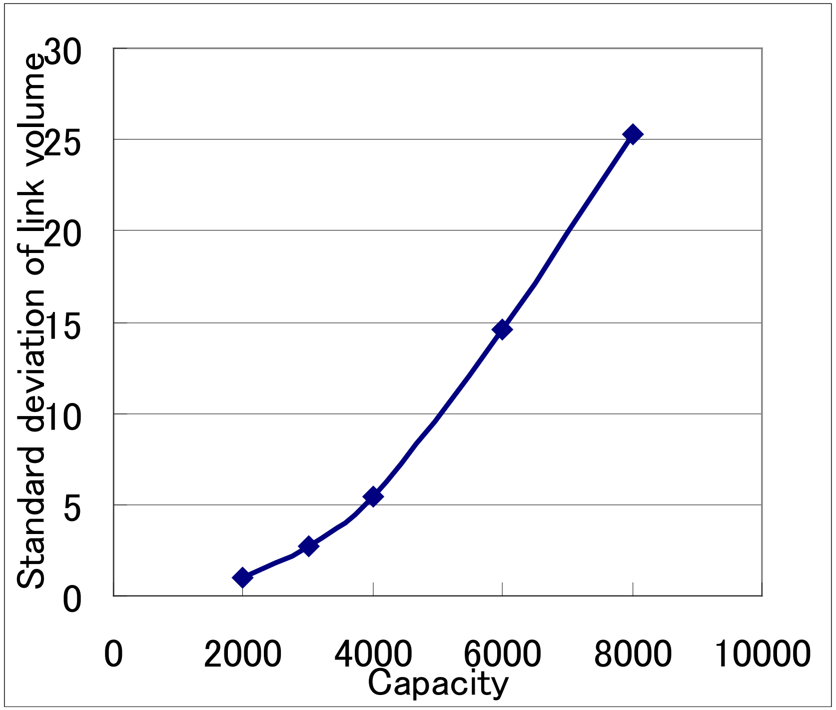
- Congestion negates the fluctuation of link volume, but increases variation of travel time because even the small change in volume causes large change in travel time.

Effects of demand and capacity

s.d. of link volume by demand



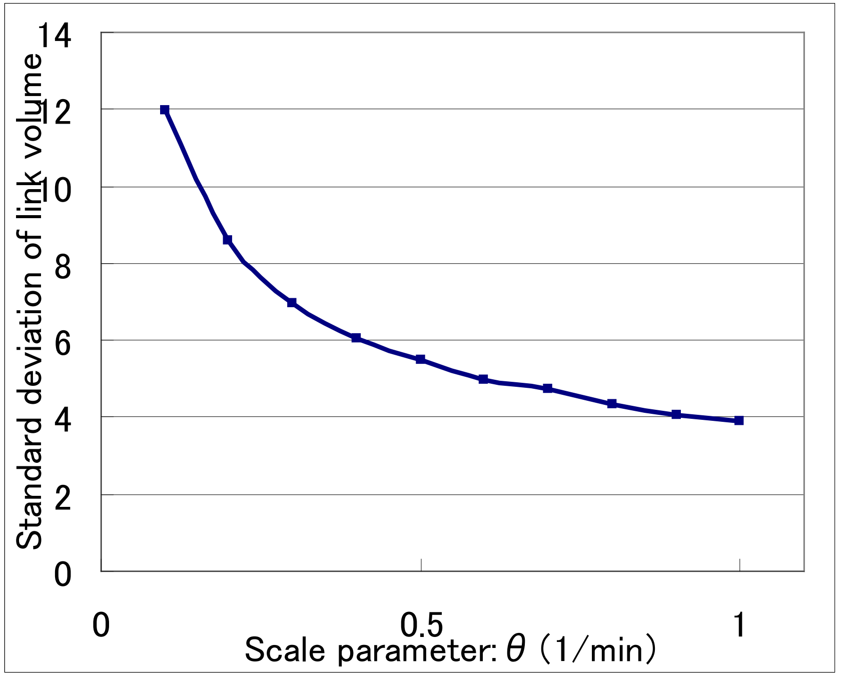
s.d. of link volume by capacity



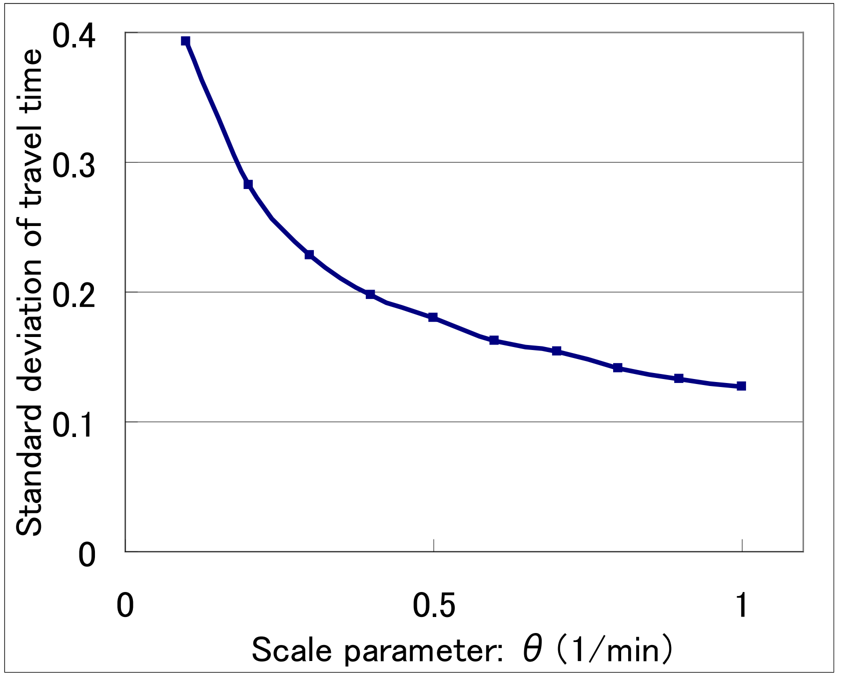
- Effect of capacity is opposite to that of demand as expected.

Effect of scale parameter

s.d. of link volume



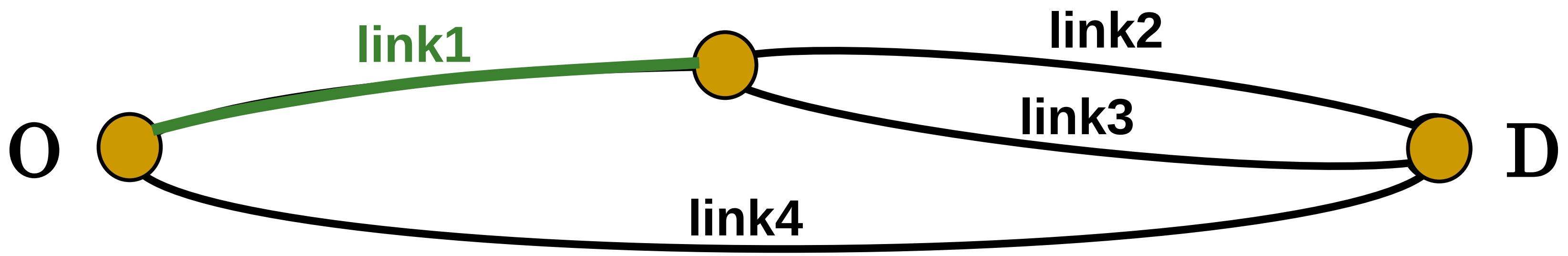
s.d. of travel time



- Both fluctuations of link flow and travel time decrease according to the scale parameter.

Case 2: 1 OD 3 routes with overlapping

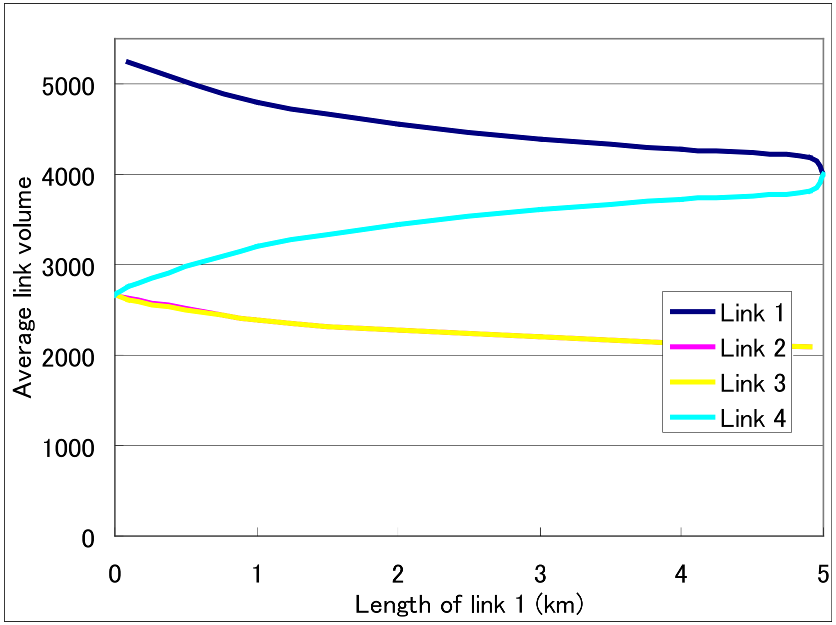
- θ = 0.1, OD length = 5km, demand = 8000, C = 4000



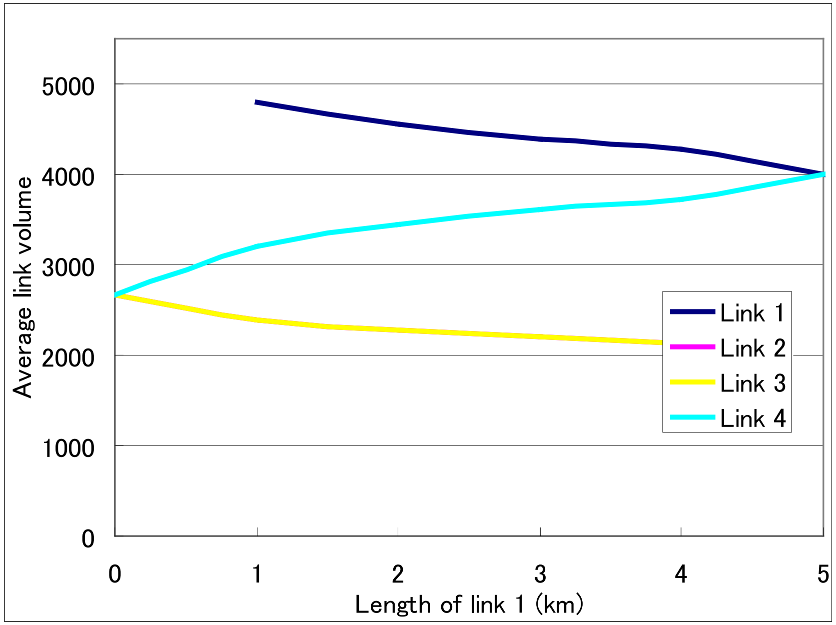
- Examine the effect of the length of link 1

Consistency between CSUE and SUE

Average link volume of CSUE



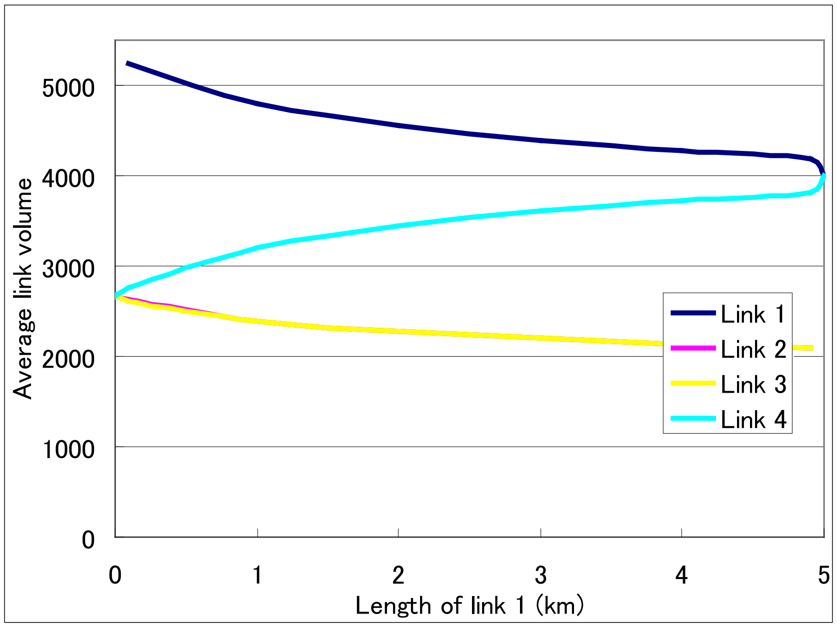
SUE



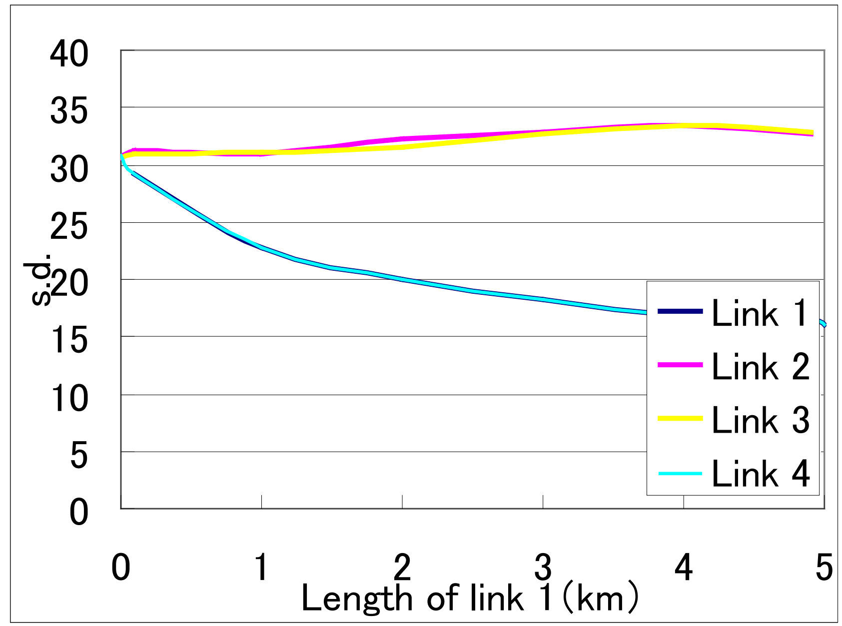
- Average link flow of CSUE is consistent with link flow of SUE.

Effect on link volume

Average link volume

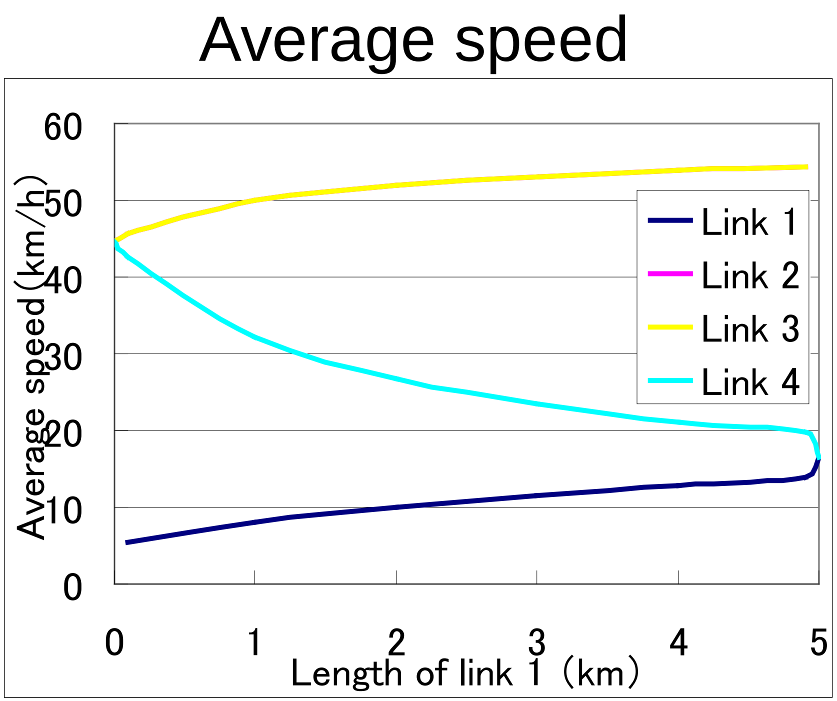


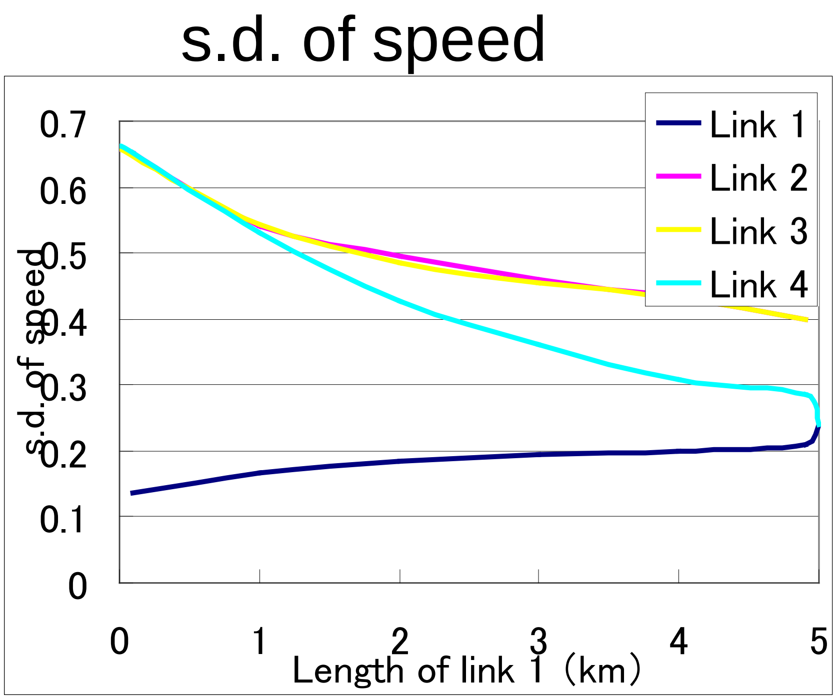
s.d. of link volume



- Average link volumes of link 1 and 4 are different, but s.d. are identical because of the negative perfect correlation between the link volumes of the two links.

Effect on speed



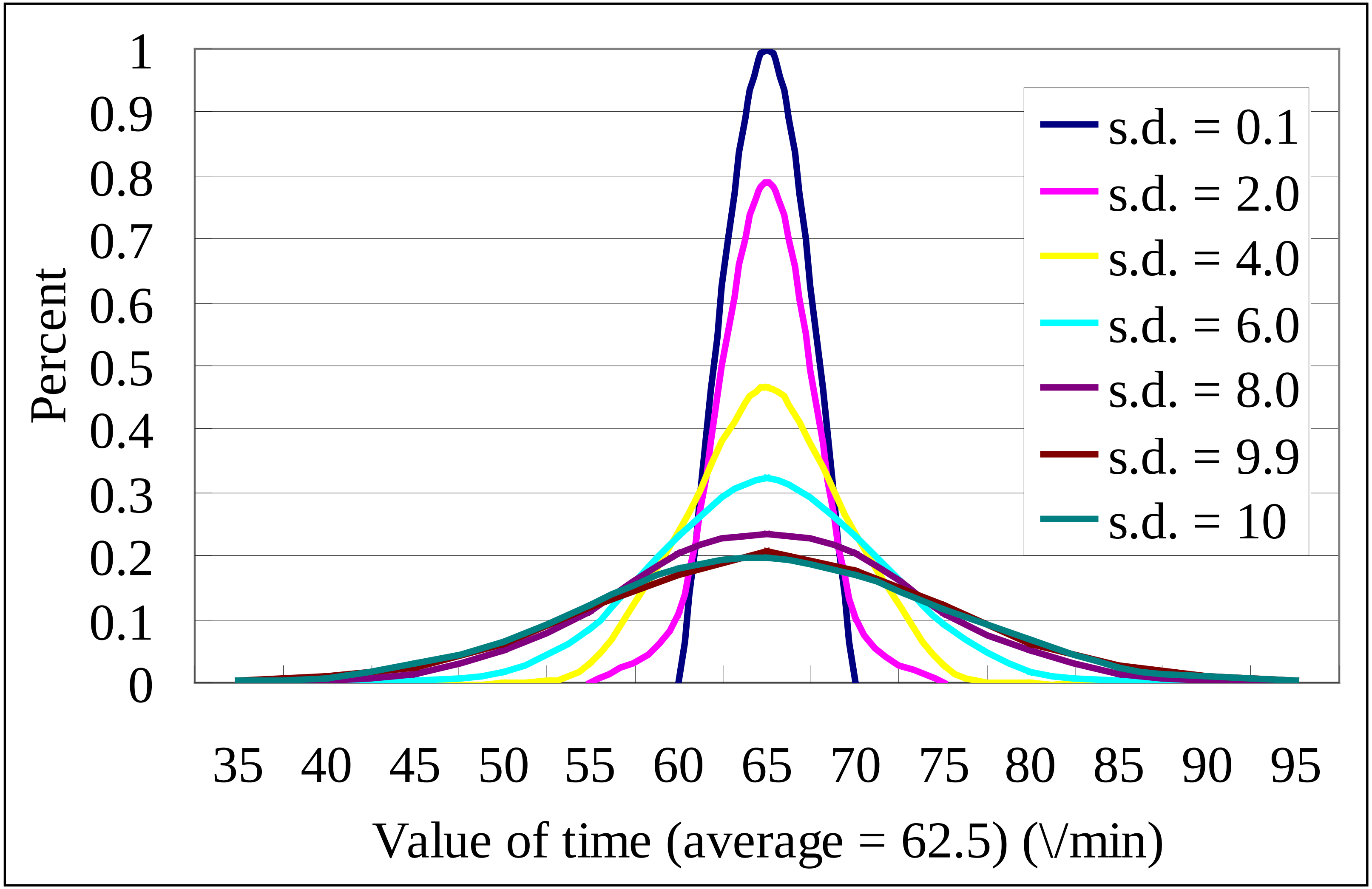


Case 3: Heterogeneous value of time

- Heterogeneity in value of time across travellers causes no problem in CSUE.
- $\theta = 0.1$, l = 5km, demand = 8000, C = 4000

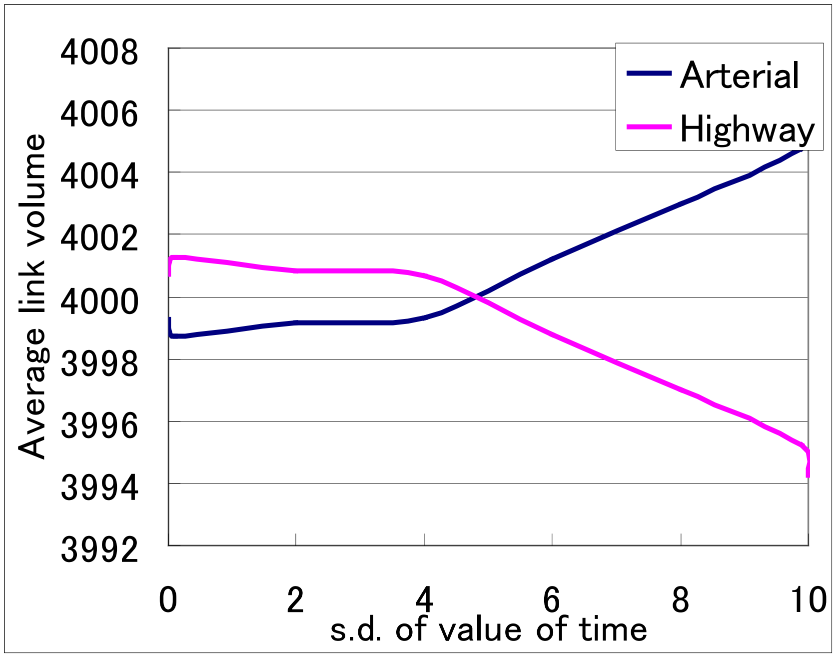
$$t_{highway}(x) = 5_{(km)} \times (60/100)_{(m/km)} \left\{ 1 + 2.62 \left(\frac{x}{4000} \right)^5 \right\} + \frac{450}{VOT}$$

Distribution of value of time

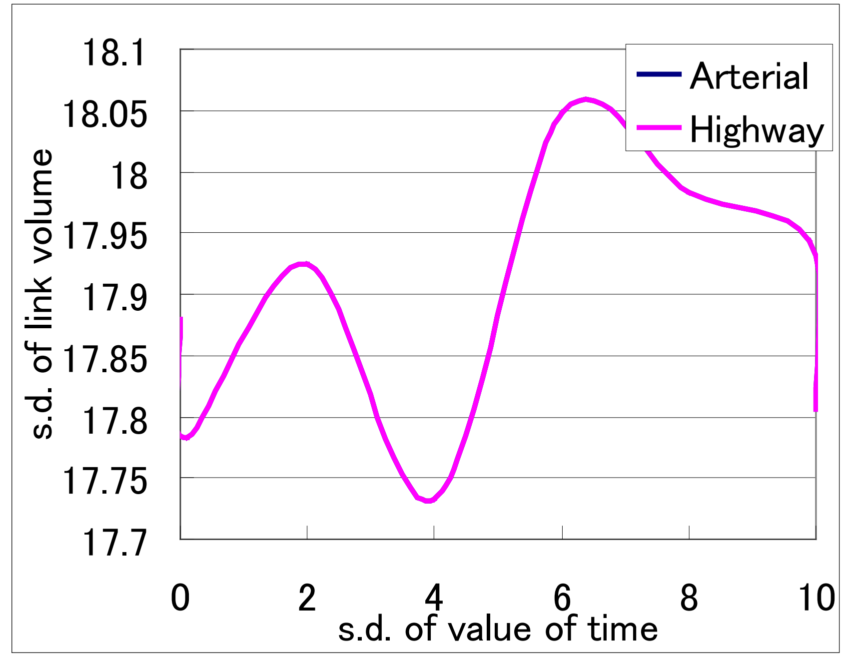


Effect on link volume

Average link volume



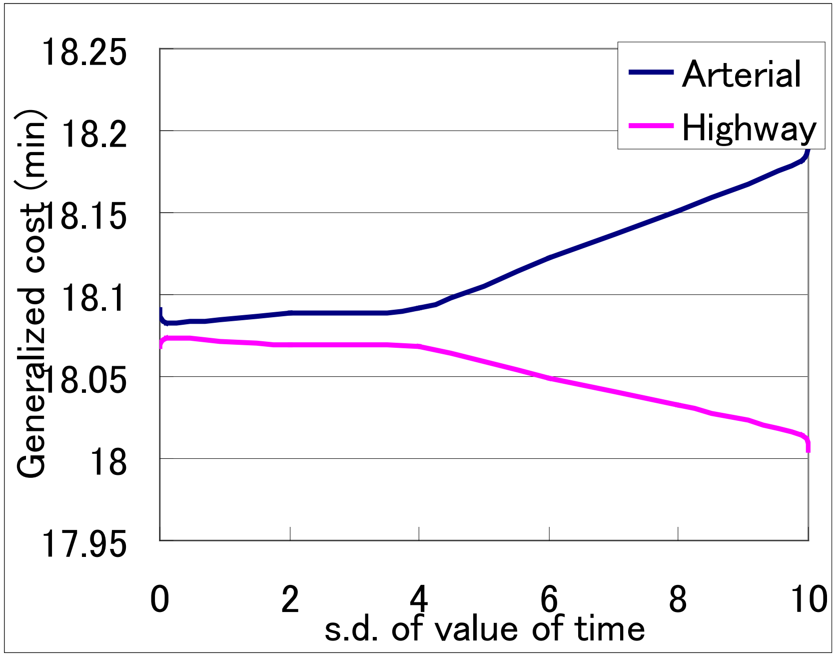
s.d. of link volume



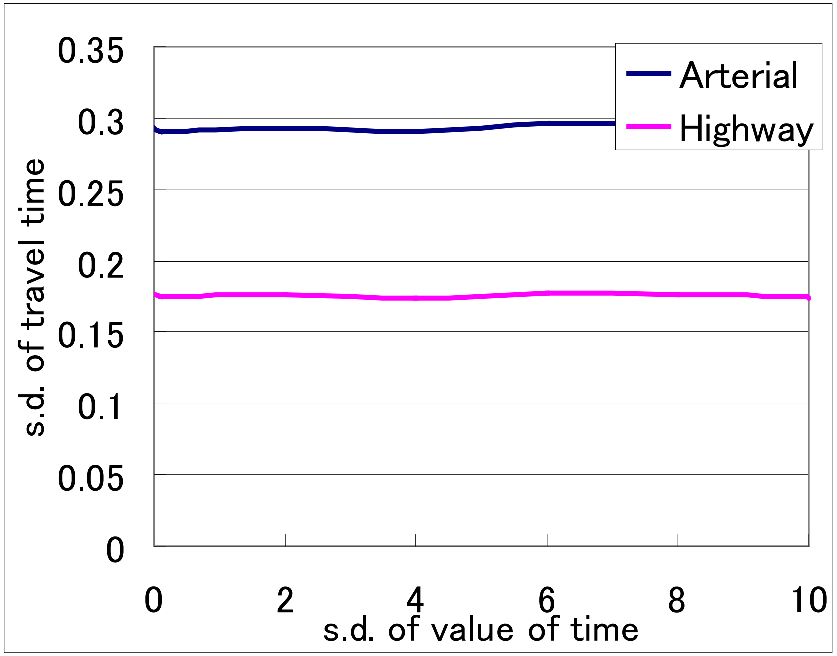
- Standard deviation of link volume looks fluctuated across standard deviation of VOT, but fluctuation is small.

Effect on generalized cost (min.)

Average generalized cost



s.d. of generalized cost



- Something wrong with the program? Need verification?

Future research

- Link nested logit model to investigate correlation of traffic flows of overlapping routes
- Extension to elastic demand (integrated mode choice and assignment)
  - Variation of traffic flow caused by both mode choice and route choice
- Extension to dynamic user equilibrium
  - No problem with uniqueness of the equilibrium
- Application to real networks

References

- Hazelton, M.L. , S. Lee and J.W. Polak (1996) Stationary states in stochastic process models of traffic assignment : a Markov chain Monte Carlo approach, Proceedings of the 13th ISTTT, 341-357.
- Hazelton, M.L. (1998) Some remarks on stochastic user equilibrium, Transportation Research Part B, Vol. 32, 101-108.